ON PLANE NOZZLE DESIGN

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A new particular solution has been obtained for the approximate system of equations of motion of a gas, which is close to the Chaplygin system of equations over a large transonic range of variation of velocity. This solution can be used for nozzle design.

The Chaplygin system of equations, in canonical form, can be represented as:

$$\frac{\partial \varphi}{\partial \vartheta} = \sqrt{K}\frac{\partial \psi}{\partial s}, \qquad \frac{\partial \varphi}{\partial s} = -\sqrt{K}\frac{\partial \psi}{\partial \vartheta} \tag{1}$$

where ϕ and ψ are velocity potential and stream function respectively, $\sqrt{K}$ and s are known functions of the relative velocity λ, and

$$\sqrt{K} = \sqrt{\frac{1-\lambda^2}{(1-\lambda^2/h^2)^{h^2}}}$$

$$s = \int_1^\lambda \sqrt{\frac{1-\lambda^2}{1-\lambda^2/h^2}}\frac{d\lambda}{\lambda} \tag{2}$$

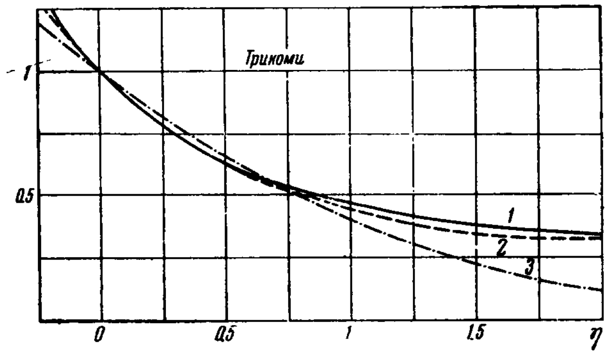
$$\left(h^2 = \frac{\varkappa+1}{\varkappa-1}\right)$$

and θ is the angle between the velocity vector and the abscissa in the plane of gas flow (x, y). In the neighborhood of $\lambda = 1$ we have

$$\sqrt{K} = A_0 s^{1/3}, \qquad A_0 = -3^{1/3}\left(\frac{\varkappa+1}{2}\right)^{\frac{\varkappa+2}{3(\varkappa-1)}}$$

Let us now put system (1) in terms of new independent variables η, α, using the formulas

$$s = -\frac{2}{3}\eta^{3/2}, \qquad \vartheta = -\eta\alpha - \frac{1}{3}\alpha^3 \tag{3}$$



As a result of some simple transformations we obtain

$$\frac{\partial\varphi}{\partial\eta}=\left(\frac{2}{3}\right)^{1/3}A_0F(\eta)\left(\frac{\partial\psi}{\partial\alpha}-\alpha\,\frac{\partial\psi}{\partial\eta}\right),\qquad \frac{\partial\varphi}{\partial\alpha}=\left(\frac{2}{3}\right)^{1/3}A_0F(\eta)\left(\alpha\,\frac{\partial\psi}{\partial\alpha}-(\eta+\alpha^2)\,\frac{\partial\psi}{\partial\eta}\right) \tag{4}$$

or we get one equation for the stream function

$$\frac{\partial^2\psi}{\partial\alpha^2}-2\alpha\,\frac{\partial^2\psi}{\partial\alpha\partial\eta}+(\eta+\alpha^2)\,\frac{\partial^2\psi}{\partial\eta^2}-\frac{F'(\eta)}{F(\eta)}\left(\alpha\,\frac{\partial\psi}{\partial\alpha}-(\eta+\alpha^2)\,\frac{\partial\psi}{\partial\eta}\right)=0 \tag{5}$$

where

$$F(\eta)=\frac{\sqrt{K}}{A_0 s^{1/3}} \tag{6}$$

For $F = 1$, equation (5) has the particular solution $\psi = \alpha$, which corresponds to Fal'kovich's result [1]. It should be mentioned that the exact function $F(\lambda)$ differs significantly from unity over a wide range of velocity variation λ in the neighborhood of sonic velocity. To get a more accurate result we will look for some particular solution in the form

$$\psi=f(\alpha)\,[1+aJ(\eta)]\qquad\left(J(\eta)=\int_0^\eta\frac{d\eta}{F(\eta)}\right) \tag{7}$$

where a is an arbitrary constant. By putting expression (7) into equation (5) and separating the variables, we obtain

$$\frac{f''(\alpha)}{\alpha f'(\alpha)}=\frac{F'(\eta)}{F(\eta)}+\frac{2a}{F(\eta)\,[1+aJ(\eta)]}=n \tag{8}$$

where n is an arbitrary constant. From this we obtain

$$F(\eta)=\frac{a}{An}\,e^{n\eta}(A-1+e^{-n\eta})^2,\quad f(\alpha)=c_0+c_1\int_0^\alpha e^{1/2\,n\alpha^2}\,d\alpha \tag{9}$$

where A, c_0, c_1 are arbitrary constants of integration.

From the condition that function (9) and its differential coincide at point $\eta = 0$ with the corresponding exact values of function (6), we obtain

$$a = \frac{n}{A}, \qquad n = \frac{A}{A-2} F'(0) \tag{10}$$

In the figure curve *1* represents the exact function (6), curve *2* is function (9) for condition (10) and $A = 0.096$. Curve *3* represents the relation between λ and η. When $c_0 = 0$, solution (7) represents a symmetrical nozzle, and satisfies the well-known condition of the persistence of subsonic flow within the supersonic region at the line of transition [2].

BIBLIOGRAPHY

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